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STABILIZATION OF THE PROGRAMMED MOTIONS OF A RIGID BODY WITH UNCERTAINTY IN THE PARAMETERS OF THE EQUATIONS OF DYNAMICS†

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A control scheme is proposed to guarantee the asymptotic stability of a given programmed motion of a rigid body rotating about a fixed point. The body is controlled by means of a couple of reactive forces, or the control action is created by rotating flywheels. The inertial parameters and angular momentum of the system are estimated while the motion is in progress. The control is synthesized by expressing the equations of dynamics in a form that is linear in the parameter vector and by using the passivity property of the dynamical object. A law is proposed for the control and for adjusting the parameters that guarantees the asymptotic stability of the motion and, for programmed motions that satisfy the condition of non-vanishing action, guarantees the convergence of the vector of adjusted parameters to its true value. The domain in phase space for which exponential stabilization is achieved is determined. © 1997 Elsevier Science Ltd. All rights reserved.

An analogous problem has been considered previously [1]; the control law obtained there for the rotary motion of an unmanned aircraft requires the computation of complicated non-linear expressions, including the Jacobian of the kinematic relations and its derivatives, which are not well-conditioned for rotations close to $\pm\pi$.

Unlike those results, the algorithms proposed below have no singularities anywhere in phase space.

A dynamical control of a rigid body for certain inertial parameters was constructed in [2]. Recurrent algorithms for setting up the equations of dynamics have been used to synthesize a control that ensures stabilization of the programmed motion of a multiple chain of rigid bodies [3].

1. DYNAMICAL MODEL OF THE SYSTEM

We will consider a rigid body P_0 with a fixed point O at its centre of mass. Together with P_0 we consider a system of coordinates $Oxyz$ attached to the body and a triple of mutually orthogonal unit vectors c_1, c_2, c_3 occupying a fixed position in $Oxyz$. A programmed motion is defined by a triple of mutually orthogonal unit vectors s_1, s_2, s_3 rotating as a rigid body at angular velocity ω_d relative to the absolute system of coordinates $O\xi\eta\zeta$.

Rotary motion of the body is described by Euler's equations of dynamics

$$M\dot{\omega} + \omega \times M\omega = u \tag{1.1}$$

where ω is the angular velocity of the controlled body and $u \in R^3$ is a vector of control torques created by couples of reactive forces.

If P_0 is controlled by means of rotating symmetric flywheels $P_1, \ldots, P_m$, then, when there are no external forces, the equations of motion of the system of bodies may be written as follows [2, 4]:

$$M\dot{\omega} + \omega \times R^t L_0 = u, \quad L_0 = \text{const}, \quad M = \theta_0 - \sum_{i=1}^m C_i b_i b_i^t, \quad u = \sum_{i=1}^m b_i \tau_i \tag{1.2}$$

where θ_0 is the inertia tensor of the system of bodies, b_i is a unit vector along the axis of rotation of the ith flywheel (the superscript t denotes transposition), C_i is the moment of inertia of the ith flywheel about its axis of rotation, L_0 is the angular momentum of the system of bodies in the system of coordinates

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$O\xi\eta\zeta$, $R \in SO(3)$ is the matrix of a rotation transforming the axes $O\xi\eta\zeta$ into axes $Oxyz$, and τ_i is a control torque applied to the axis of the ith flywheel.

Henceforth we will use a common notation for (1.1) and (1.2)

$$M\dot{\omega} + \omega \times N(\omega, L_0) = u \tag{1.3}$$

and consider the equation along with the Poisson kinematic equations

$$\dot{s}_i = -(\omega - \omega_d) \times s_i \quad (i = 1, 2, 3) \tag{1.4}$$

The inertial parameters and angular momentum of the dynamical object (1.3), (1.4) are unknown. It is required to determine a control torque u and adjustment law for the control parameters of the system so that the axes c_1, c_2, c_3 attached to the body will stabilize in the direction of the corresponding fixed axes s_1, s_2, s_3 and the angular velocity ω will tend to ω_d.

To determine the control law we will use the following property of the dynamical object (1.3).

1. The equation of dynamics (1.3) can be expressed in a form that is linear in the vector of unknown parameters $\theta \in R^p$

$$M\dot{\omega}_r + \omega_r \times N(\omega, L_0) = Y(\omega, \omega_r, \dot{\omega}_r)\theta \tag{1.5}$$

where ω_r, $\dot{\omega}_r \in R^3$ are arbitrary vectors, the vector θ contains the unknown components of the inertia matrix and the angular momentum, and Y is a known $3 \times p$ matrix-valued function.

If the control is established by means of a couple of reactive forces and $Oxyz$ are the principal central axes of inertia of P_0, then $N(\omega, L_0) = M\omega$, $\theta = (A, B, C)^t$ and the 3×3 matrix Y is

$$Y = \begin{Vmatrix} \dot{\omega}_{r1} & -\omega_2\omega_{r3} & \omega_{r2}\omega_3 \\ \omega_1\omega_{r3} & \dot{\omega}_{r2} & -\omega_{r1}\omega_3 \\ -\omega_1\omega_{r2} & \omega_{r1}\omega_2 & \dot{\omega}_{r3} \end{Vmatrix}$$

The following passivity property of the dynamical object is useful in finding the Lyapunov function used in synthesizing the control.

2. The differential equation (1.3) defines a passive mapping $u \to \omega$, that is

$$\int_0^t u^t \omega d\tau \geqslant -\gamma^2 \text{ for all } T > 0 \text{ and some } \gamma \in R. \tag{1.6}$$

To prove inequality (1.6), we find

$$\int_0^T \omega^t (M\dot{\omega} + \omega \times N(\omega, L_0)) d\tau = \frac{1}{2}(\omega^t(T)M\omega(T) - \omega^t(0)M\omega(0)) \geqslant -\gamma^2$$

We now introduce the normed spaces of square-integrable functions

$$L_2^n(R_+) = \left\{ x: R_+ \to R^n \left| \left(\int_0^\infty |x(t)|^2 dt \right)^{1/2} < \infty \right. \right\}$$

and bounded functions

$$L_\infty^n(R_+) = \left\{ x: R_+ \to R^n \left| \sup_{t \in [0,\infty)} |x(t)| < \infty \right. \right\}$$

where R_+ is the set of non-negative real numbers.

Lemma 1. Let $x: R_+ \to R^n$ be a real piecewise-smooth vector-valued function such that $x \in L_2^n \cap L_\infty^n$ and $\dot{x} \in L_\infty^n$. Then $x \to 0$ as $t \to \infty$.

Proof. It follows from the assumptions of the lemma that the function $\varphi = x^t x$: $R_+ \to R^1$ satisfies the conditions

$$\dot{\varphi} = 2x^t\dot{x} \in L_\infty \text{ and } \int_0^\infty \varphi dt = \int_0^\infty x^t x dt \leq \text{const} \tag{1.7}$$

Consequently, φ is in L_1 and is also uniformly continuous on R_+. Therefore [5] $\varphi \to 0$ as $t \to \infty$ and so also $x \to 0$ as $t \to \infty$.

2. SYNTHESIS OF THE DYNAMIC CONTROL WITH PARAMETER ADJUSTMENT

We now define vector-valued functions characterizing the tracking errors of the programmed motion

$$e = \sum k_i s_i \times c_i, \quad \omega_r = \omega_d - \Lambda e, \quad v = \omega - \omega_r \tag{2.1}$$

The vector $e \in R^3$ characterizes the orientation error of the rigid body; k_i are pairwise distinct positive numbers ($i = 1, 2, 3$). The vector $v \in R^3$ takes the orientation error and angular velocity error into account: $v = \omega_e + \Lambda e$, where $\omega_e = \omega - \omega_d$ and $\Lambda > 0$. In all cases summation is from $i = 1$ to $i = 3$.

Let $\theta_r = R^p$ be a parameter-estimate vector for the object and $\theta_e \in R^p$, $\theta_e = \theta_r - \theta$ the vector of parameter-estimate errors. It is assumed that the programmed motion satisfies the desired smoothness conditions $\omega_d \in C^1(R_+)$.

The problem is to define a control $u(t)$ and a parameter-adjustment law so that the following control objective is attained

$$\lim_{t\to\infty} \omega = \omega_d, \quad \lim_{t\to\infty} s_i = c_i, \quad i = 1, 2, 3 \tag{2.2}$$

We will also define an auxiliary control objective

$$\lim_{t\to\infty} \omega = \omega_d, \quad \lim_{t\to\infty} e = 0 \tag{2.3}$$

which determines the convergence of the motions of the body to motions $\omega = \omega_d$, $s_1 = \pm c_1$, $s_2 = \pm c_2$, $s_3 = s_1 \times s_2$, one of which corresponds to the programmed trajectory.

Lemma 2. For all $T > 0$

$$\int_0^T \omega_e^t e d\tau \geq -\gamma^2, \quad \gamma^2 = 2\sum k_i \tag{2.4}$$

Proof. Substitution of (2.1) into (2.4) gives

$$\int_0^T \omega_e^t \sum_{i=1}^{i=3} k_i s_i \times c_i d\tau = \int_0^T \sum_{i=1}^{i=3} k_i c_i^t (\omega_e \times s_i) d\tau = -\int_0^T \sum_{i=1}^{i=3} k_i c_i^t \dot{s}_i d\tau = -\sum_{i=1}^{i=3} k_i c_i^t s_i \Big|_0^T \geq -\gamma^2$$

Lemma 3. If $v \in L_2^3$, then $e, \omega_e \in L_2^3$.

Proof. Using (2.1) and Lemma 2, we find that

$$\int_0^T \omega_e^2 d\tau + \int_0^T e^2 d\tau = \int_0^T v^2 d\tau - 2\Lambda \int_0^T \omega_e^t e d\tau \leq \int_0^\infty v^2 d\tau + 2\Lambda\gamma^2 < \infty \tag{2.5}$$

Now let $T \to \infty$.

Consider the following Lyapunov function

$$V = [v^t M v + \theta_e^t \Gamma^{-1} \theta_e]/2 \tag{2.6}$$

where Γ is the symmetric positive-definite 3×3 matrix of feedback coefficients.

Differentiation of Eq. (2.6) along the trajectories of the system yields the control and parameter-adjustment law

$$u = M_r\dot{\omega}_r + \omega_r \times N(\omega, L_{0r}) - K_d v \tag{2.7}$$

$$\dot{\theta}_r = -\Gamma Y'(\omega, \omega_r, \dot{\omega}_r)v \tag{2.8}$$

where K_d is the symmetric positive-definite 3×3 matrix of feedback coefficients and M_r and L_{0r} are estimates for the inertia tensor and angular momentum vector.

Theorem 1. Consider the motion of the rigid body (1.3), (1.4) subject to a control (2.7) and parameter-adjustment law (2.8). Then the bounded programmed trajectories ω_d, $\dot{\omega}_d \in L^3_\infty(R_+)$ achieve the auxiliary control objective (2.3) and the motions of the rigid body tend to asymptotically stable motions $\omega = \omega_d$, $s_1 = \pm c_1, s_2 = \pm c_2, s_3 = s_1 \times s_2$. The adjusted-parameter vector θ_r and control u remain bounded.

Proof. Differentiation of Eq. (2.6), taking (1.3) and (1.4) into account, yields

$$\dot{V} = v'M(\dot{\omega} - \dot{\omega}_r) + \theta_e^t\Gamma^{-1}\dot{\theta}_e = v'(u - \omega_r \times N(\omega, L_0) - M\dot{\omega}_r) + \theta_e^t\Gamma^{-1}\dot{\theta}_e$$

Using the linear representation (1.5) and taking (2.7) and (2.8) into account, we obtain

$$\dot{V} = -v'K_d v \tag{2.9}$$

Here we have used the equality $\dot{\theta}_e = \dot{\theta}_r$, since $\dot{\theta} = 0$. It follows from (2.6) and (2.9) that

$$v \in L_2^3 \cap L_\infty^3, \quad \theta_e \in L_\infty^p$$

Using Lemma 3, we obtain ω_e, $e \in L_2^3$, and it follows from the fact that e and v are bounded that $\omega_e \in L^3_\infty$.

Differentiation of Eq. (2.1) gives

$$\dot{e} = -\sum k_i(\omega_e \times s_i) \times c_i$$

that is, the boundedness of ω_e implies that of $\dot{e}$. Consequently, by Lemma 1, $e \to 0$ as $t \to \infty$.

We now prove that the vector v tends to zero. Since $\omega_d \in L^3_\infty$, the fact that ω_e is bounded implies that ω is bounded. Taking into account that θ_e, ω, ω_d, $\dot{e}$, are bounded, we deduce from (2.7) that the control u is bounded. It then follows from (1.3) that $\dot{\omega} \in L^3_\infty$. Since $\dot{\omega}$ is bounded, the same is true of $\dot{v}$ and so, by Lemma 1, $v \to 0$ as $t \to \infty$.

Since v and e tend to zero as $t \to \infty$, it follows that the motions of the rigid body tend to asymptotically stable motions $\omega = \omega_d$, $s_1 = \pm c_1$, $s_2 = \pm c_2$, $s_3 = s_1 \times s_2$, one of which corresponds to the programmed trajectory.

As an example, let us consider the problem of stabilizing a given position of a rigid body. Setting $\omega_d = \dot{\omega}_d = 0$ in (2.7), we obtain

$$u = -K_d\omega - K_d \sum k_i s_i \times c_i - \Lambda Y(\omega, e, \dot{e})\theta_r \tag{2.10}$$

This control differs from the stabilizing control of [2, p. 425] by dynamical terms proportional to the estimated parameters of the object, which improve the convergence of the orientation error to zero.

3. EXPONENTIAL STABILIZATION OF THE MOTION

To obtain exponential stabilization of a programmed motion and convergence of the estimated parameters of the object to their real values, we replace the Lyapunov function (2.6) by

$$V = 1/2[v'Mv + \theta_e^t\Gamma^{-1}\theta_e + K_p\sum k_i(c_i - s_i)^2 + \|Mv - \varepsilon - \varphi^t\theta_e\|^2 \tag{3.1}$$

where $\varepsilon \in R^3$ is an auxiliary vector and φ is a $p \times 3$ matrix-valued function.

An additional term analogous to the last term in (3.1) was used in [6] for the exponential stabilization of the motion of a manipulator with rigid arms.

Differentiation of Eq. (3.1) gives the following control and parameter-adjustment law

$$u = Y_c\theta_r - K_p e - \alpha \upsilon, \quad \dot{\theta}_r = -\Gamma Y_c^t \upsilon + \lambda_c \Gamma \varphi \varepsilon \tag{3.2}$$

$$\dot{\varepsilon} = -\lambda_c \varepsilon - \alpha \upsilon - K_p e + \varphi^t \Gamma Y_c^t \upsilon - \lambda_c \varphi^t \Gamma \varphi \varepsilon, \quad \dot{\varphi} = -\lambda_c \varphi + Y_c^t$$

where $\lambda_c > 0$ and $\alpha > \lambda_c(\sigma_M)^2$, σ_M being the largest eigenvalue of M; the $3 \times p$ matrix-valued function Y_c is defined by the equality

$$Y_c(\omega, \omega_r, \dot{\omega}_c)\theta = M\dot{\omega}_c + \omega_r \times N(\omega, L_0), \quad \dot{\omega}_c = \dot{\omega}_r - \lambda_c \upsilon \tag{3.3}$$

The matrix Γ in (3.2) is adjusted by using the method of least squares [7]

$$\dot{\Gamma}^{-1} = \frac{\lambda_c}{2}(\varphi\varphi^t - \lambda(t)\Gamma^{-1}), \quad \Gamma(0) = \Gamma^t(0) > 0, \quad \|\Gamma(0)\| \leqslant k_0; \quad \lambda(t) = \lambda_0\left(1 - \frac{\|\Gamma\|}{k_0}\right) \tag{3.4}$$

with a time-dependent parameter $\lambda(t)$.

The adjustment law (3.4) has the following properties: (a) for any $t \geqslant 0$ we have inequalities $\lambda(t) \geqslant 0$ and $\Gamma(t) \leqslant k_0 I$; (b) for programmed motions with non-vanishing action, i.e. for which positive numbers β_1, β_2 and δ such that

$$\beta_1 I_3 \leqslant \int_t^{t+\delta} \varphi^t(\tau)\varphi(\tau)d\tau \leqslant \beta_2 I_3 \tag{3.5}$$

for any $t \geqslant 0$, a positive number $\lambda_1 \geqslant 0$ exists such that $\lambda(t) \geqslant \lambda_1$ for any $t \geqslant 0$.

If the variable $\lambda(t)$ can be separated from zero, one can ensure that the parameter-estimate vector θ_r will converge to its true value.

The control (3.2)–(3.4) is composite [7], since the parameter-adjustment operation uses both information about the tracking error of the programmed motion and the predicted value of the error.

Theorem 2. Consider the motion of a rigid body (1.3), (1.4) with the control and parameter-adjustment law (3.2)–(3.4). Then, if the programmed motions ω_d, $\dot{\omega}_d \in L^3_\infty(R_+)$ are bounded, the auxiliary control objective (2.3) will be achieved.

Moreover, if the programmed motion is such that the non-vanishing action condition (3.5) holds, then

(a) the vector θ_r tends to the true value of the object parameters;

(b) the vector ε tends to zero, $\varepsilon \to 0$;

(c) all steady motions $\omega = \omega_d$, $s_1 = \pm c_1$, $s_2 = \pm c_2$, $s_3 = s_1 \times s_2$, $\theta_e = \varepsilon = 0$ except the programmed motion are unstable, and the Lyapunov function (3.1) tends exponentially to zero in the domain

$$D: \{s : e^t e > \Lambda^{-1} V_\delta\}, \quad \left(V_\delta = \sum_{i=1}^{3} k_i (c_i - s_i)^2\right) \tag{3.6}$$

Proof. Differentiation of the first term in (3.1), taking (1.3), (1.5) and the first equation of (3.2) into consideration, yields

$$\begin{aligned} T_1 &= \upsilon^t(u - \omega \times N(\omega, L_0) - M\dot{\omega}_r) = \upsilon^t(u - \omega_t \times N(\omega, L_0) - M\dot{\omega}_r) = \\ &= \upsilon^t(u - Y_c\theta - \lambda_c M\upsilon) = -\lambda_c \upsilon^t M \upsilon + \upsilon^t(u - Y_c\theta_r + Y_c\theta_e) = \\ &= -\lambda_c \upsilon^t M\upsilon - \alpha\upsilon^t\upsilon - K_p \upsilon^t e + \upsilon^t Y_c \theta_e \end{aligned} \tag{3.7}$$

Differentiation of the second and third terms of (3.1) gives

$$T_2 + T_3 = \theta_e^t \Gamma^{-1}\dot{\theta}_e + \frac{\lambda_c}{4}\left(\|\theta_e\|^2_{\varphi\varphi^t} - \lambda(t)\|\theta_e\|_{\Gamma^{-1}}\right) + K_p \omega_e^t e \tag{3.8}$$

where $\| X \|_A$ is the quadratic norm generated by a symmetric matrix A.

Differentiation of the last term in (3.1), taking (3.2) into consideration, gives [6]

$$T_4 = (Mv - \varepsilon - \varphi'\theta_e)'(-\lambda_c Mv + Y_c\theta_e - K_p e - \alpha v - \dot{\varepsilon} - \dot{\varphi}'\theta_e - \varphi'\dot{\theta}_e) =$$
$$= \frac{\lambda_c}{2}\|Mv - \varepsilon - \varphi'\theta_e\|^2 - \frac{\lambda_c}{2}\|Mv - \varepsilon\|^2 + \lambda_c(Mv - \varepsilon)'\varphi'\theta_e - \frac{\lambda_c}{2}\|\theta_e\|^2_{\varphi\varphi'} \leqslant$$
$$\leqslant -\frac{\lambda_c}{2}\|Mv - \varepsilon - \varphi'\theta_e\|^2 - \frac{\lambda_c}{2}\|\theta_e\|^2_{\varphi\varphi'} - \lambda_c\varepsilon'\varphi'\theta_e + \frac{\lambda_c}{2}\sigma_M\left(\mu\|v\|^2 + \frac{1}{\mu}\|\theta_e\|^2_{\varphi\varphi'}\right) \tag{3.9}$$

Setting $\mu = 2\sigma_M$ in (3.9) and adding equations (3.7)–(3.9) together, we obtain

$$\dot{V} \leqslant -\lambda_c v' Mv - \frac{\lambda_c}{2}\|Mv - \varepsilon - \varphi'\theta_e\|^2 - \frac{\lambda_c}{4}\lambda(t)\|\theta_e\|^2_{\Gamma^{-1}} - K_p\Lambda e'e \tag{3.10}$$

The first part of the theorem now follows from (3.1) and (3.10).

We will now show that the parameter-estimate vector tends to its true value: $\theta_e \to 0$, provided condition (3.5) holds. Taking (3.5) into account, we deduce from (3.1) and (3.10) that $\theta_e \in L_2^p \cap L_\infty^p$.

We will now prove that $\dot{\theta}_e$ is bounded because of the second equality of (3.2). Hence $\theta_e \to 0$ as $t \to \infty$.

Now $\varepsilon \to 0$, since $\eta = Mv - \varepsilon - \varphi'\theta_e \to 0$, which in turn follows from $\eta \in L_2^3 \cap L_\infty^3$ (3.10) and the boundedness of the derivative $\dot{\eta} \in L_\infty^3$ (3.2).

Using the convergence of the parameter-estimate vector to its true value, we will now show that all steady motions other than the programmed motion are unstable in Lyapunov's sense.

We will show, for example, that the steady motion $\omega = \omega_d$, $s_1 = -c_1$, $s_2 = c_2$, $s_3 = -c_3$, $\theta_e = \varepsilon = 0$ is unstable. To that end, take the function

$$V_c = [v'Mv + \theta_e'\Gamma^{-1}\theta_e - k_1(c_1 + s_1)^2 + k_2(c_2 - s_2)^2 - k_3(c_3 + s_3)^2] + \|Mv - \varepsilon - \varphi'\theta_e\|^2/2 \tag{3.11}$$

which is bounded in the domain $V_c < 0$, exists in an arbitrarily small neighbourhood of the steady motion $\omega = \omega_d$, $s_1 = -c_1$, $s_2 = c_2$, $s_3 = -c_3$, $\theta_e = \varepsilon = 0$ for all $t \geqslant t_0$ and has a negative-definite derivative (3.10). Now apply Chetayev's instability theorem [8].

The proof that the other steady motions are unstable is analogous. In the domain (3.6), inequality (3.10) may be written as follows:

$$\dot{V} + \gamma V \leqslant 0, \quad \gamma = \min\{\lambda_c, \lambda_c\lambda_1/2, 2\} > 0 \tag{3.12}$$

This relationship implies that the Lyapunov function tends exponentially to zero. This completes the proof of the theorem.

As an example, let us consider the problem of stabilizing the orientation of a rigid body in a given direction $s_0(t)$. Put $e = s_0 \times c_0$ and $V_\delta = (s_0 - c_0)^2$, where c_0 is a unit vector attached to the body. The first relationship for the controlling torque transforms to

$$u = M_r\dot{\omega}_d + \omega_d \times N(\omega, L_{0r}) - K_e e - K_v\omega_e - \Lambda Y(\omega, e, \dot{e})\theta_r \tag{3.13}$$
$$K_e = (K_p + \alpha\Lambda)I_3 + \lambda_c\Lambda M_r, \quad K_v = \alpha I_3 + \lambda_c M_r$$

The domain (3.6) is

$$D:\{s_0 : (s_0 c_0) > 2/\Lambda - 1\} \tag{3.14}$$

On a sphere of unit radius about the point 0, this expression defines a domain with the following property: if the end of the unit vector s_0 is within the domain, one obtains exponential stabilization of the motion provided condition (3.5) holds. If $\Lambda > 1$, D_0 encloses the equilibrium position $s_0 = c_0$, and as Λ decreases it tends to the unit sphere.

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